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Manipulating a Bose-Einstein condensate with a single photon

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Abstract. We propose a method to create macroscopic superpositions, so-called Schrödinger cat states, of different motional states of an ideal Bose-Einstein condensate. The scheme is based on the scattering of a freely expanding condensate by the light field of a high-finesse optical cavity in a quantum superposition state of different photon numbers. The atom-photon interaction creates an entangled state of the motional state of the condensate and the photon number, which can be converted into a pure atomic Schrödinger cat state by operations only acting on the cavity field. We discuss in detail the fully quantised theory and propose an experimental procedure to implement the scheme using short coherent light pulses.

PACS. 03.65.Bz Foundations, theory of measurement, miscellaneous theories (including Aharonov-Bohm effect, Bell inequalities, Berry's phase) – 03.75.Fi Phase coherent atomic ensembles; quantum condensation phenomena – 42.50.Ct Quantum description of interaction of light and matter; related experiments

1 Introduction

One of the most fundamental problems of quantum mechanics is the question why superposition states of macroscopic objects are never observed. The superposition principle which is at the heart of quantum mechanics seems to play no role in the macroscopic world. In order to illustrate this fact, Schrödinger introduced his famous gedanken experiment which leaves a cat in a superposition of dead and alive [1]. Modern measurement theory explains this by decoherence according to the coupling of the observed quantum system (the cat) to the environment. The timescales involved here are such that the decay of a macroscopic superposition is too short for an experimental observation.

The main interest has thus shifted to mesoscopic cat-like states where decoherence effects can be controlled much better and one may hope to understand the transition from quantum mechanical to classical behaviour. In quantum optics most of the work concentrates on the two different schemes where mesoscopic superposition states have been achieved experimentally: the motional degrees of freedom of a single trapped ion [2] and the light field in high-finesse optical cavities [3]. More recently, the existence of mesoscopic superposition states has also been shown by the interference of C_{60} molecules [4].

With the advent of Bose-Einstein condensation in dilute atomic gases (for recent overviews see [5–7]) one can now think of preparing multi-atom superposition states of up to one million atoms. This would allow the investigation of completely different decoherence channels such as atomic collisions and light scattering. Zoller and coworkers have proposed a system of two interacting Bose-Einstein condensates, the ground state of which can be tailored to become a Schrödinger cat state of two condensates of different internal atomic state [8].

In contrast to this, our proposal results in a mesoscopic superposition state of a condensate in two (or more) motional states. This would allow for spatial separation of the two components and hence, for instance, for the investigation of decoherence effects as a function of the distance between the condensates. The basis of our scheme is the scattering of a condensate by the light field of a high-finesse optical cavity which initially is prepared in a superposition state of different photon numbers such as a weak coherent field. The scattering process is analogous to the scattering of a single atom off a quantized light field [9] and leads to a final state which entangles the condensate external degrees of freedom and the cavity photon number, that is, a Schrödinger cat state. Erasing the "which path" information stored in the cavity then yields a condensate state where either all atoms populate one quantum state of motion or all atoms populate another. This is qualitatively different from atom optics with condensates scattered off classical light fields [10–13] where each individual atom is in a superposition state. Our system can also be considered as the atom optics analogue of a recently proposed scheme for the preparation of a cat state formed by light [14] where the role of atoms and photons has been interchanged.

Apart from the possibility of preparing cat states as discussed here, the interaction of a condensate with the light field of a cavity is an interesting subject in itself and allows numerous other applications, too. For example, this system has been proposed to manipulate quantum statistical properties of the condensate [15,16],

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for amplification of matter waves [17], and for measurements on condensates [18,19]. We will thus base our discussions on a general fully quantized model for the interaction of a non-interacting Bose gas with a single cavity mode. This allows us to investigate in detail how a coherent pump of the cavity and incoherent cavity decays effect the coherence properties of the condensate.

This paper is organized as follows. In Section 2 we discuss in an effective single-atom model the main features of the interaction of a condensate with a high-finesse cavity and show how this can be exploited to prepare an ideal cat state of the external degrees of freedom of the condensate. In Section 3 we then introduce the full many-particle model and discuss in detail all the steps and their experimental feasibility which are required for the preparation of a cat state. The comparison with actual experimental parameters shows that even with low efficiency only superposition states of two atoms can be achieved today, but an improvement of the optical cavities by one order of magnitude would allow to obtain mesoscopic superposition states of approximately ten atoms. We finally summarize our results in Section 4.

2 Idealised case

2.1 Mean-field description

We will first discuss the example of the preparation of a perfect Schrödinger cat state of a condensate under ideal conditions. This will allow us to demonstrate the basic principles of our scheme and to discuss the essential physics of the model system. We will turn to a more accurate treatment of the system in Section 3 and discuss the necessary parameter regime and the experimental feasibility in more detail.

Our system consists of N identical two-level atoms of mass M with position and momentum operators $\hat{x}_n$ and $\hat{p}_n$, $n = 1...N$, and an optical cavity which supports a single mode with wave number k described by the annihilation and creation operators a and $a^\dagger$. The cavity is assumed to be lossless and its resonance frequency far enough detuned from the atomic transition such that we are allowed to adiabatically eliminate the atomic excited states and to neglect the possibility of spontaneous decay of the atoms during the interaction time with the cavity. The full Hamiltonian for the compound N particle and cavity system is then given by

$$H = \sum_{n=1}^{N} \left[\frac{\hat{p}_n^2}{2M} + U(\hat{x}_n, t) a^\dagger a \right]. \tag{1}$$

Here $U(x,t) = U_0 \cos^2(kx) \exp(-v_z^2 t^2/w^2)$ is the sinusoidal optical potential created by a single cavity photon. U_0 is related to the atom-cavity coupling g and the detuning Δ_a of the atomic resonance frequency from the cavity frequency by $U_0 = g^2/\Delta_a$. The time dependence of the potential is according to the transverse Gaussian field intensity with waist w as the atoms travel through the cavity with a constant velocity v_z.

Note that we are dealing here with a simplified one-dimensional model which assumes that the transverse size of the condensate is small compared to the cavity waist during the interaction time. Moreover, in the Hamiltonian (1) we have neglected atom-atom interaction terms as arise from binary collisions and due to the dipole-dipole interaction mediated by the cavity mode(s) [20,21]. This is justified because, as we will see later, for practical purposes the number of atoms and hence the density is very small and because we are working in a parameter regime where spontaneous atomic decays can be neglected.

Since we neglected any cavity pumping and cavity decay, the photon number is conserved and the Hamiltonian can be split into a sum of its projections onto the m photon subspaces,

$$H = \sum_m H^{(m)} |m\rangle\langle m|, \tag{2}$$

$$H^{(m)} = \sum_{n=1}^{N} \left[\frac{\hat{p}_n^2}{2M} + U(\hat{x}_n, t) m \right]. \tag{3}$$

We can thus study the time evolution in each of these subspaces independently. In order to further simplify the discussion, we now apply the mean field approximation, that is, we assume a Bose-Einstein condensed atomic system at zero temperature where all atoms occupy the same wave function. Tracing the Hamiltonian $H^{(m)}$ over all particles but one we obtain the mean-field m-photon Hamiltonian

$$H_{\mathrm{MF}}^{(m)} = \frac{\hat{p}^2}{2M} + U(\hat{x}, t) m + (N-1) \left\langle \frac{\hat{p}^2}{2M} + U(\hat{x}, t) m \right\rangle \tag{4}$$

where the expectation value has to be taken with respect to the momentary condensate wave function $\psi(t)$. This last term gives a uniform energy shift to $H_{\mathrm{MF}}^{(m)}$ and hence does not effect the dynamics of $\psi(t)$. However, it gives a different phase to the different photon number subspaces and thus can become important for subsequent measurements on the photon state.

2.2 Preparation of a Schrödinger cat state

Let us now sketch our proposal for the generation of an atomic Schrödinger cat like state. Initially the condensate is confined in the ground state of an atomic trap which is placed slightly above a high finesse optical cavity. The cavity is prepared in a superposition of the vacuum state and the single photon Fock state,

$$\psi_{\mathrm{c}} = \frac{1}{\sqrt{2}} \left(|0\rangle + |1\rangle \right). \tag{5}$$

At time $t = 0$, the condensate is released from the trap and a momentum kick of $\hbar k$ is imparted to the atoms. Alternatively, one could think of setting up the cavity with a small angle to the horizontal axis, such that the condensate atoms have a transverse momentum of $\hbar k$ with respect to the cavity axis at the time of their interaction with the cavity field. Assuming that the spatial size of the atom trap is much larger than an optical wavelength, we can treat the initial state of the atomic wavefunction as a momentum eigenstate of one photon momentum. The compound state of the system is thus given by a direct product of the atom and cavity state

$$\psi(t=0) = |p=\hbar k\rangle \otimes \psi_{\mathrm{c}}. \tag{6}$$

Then the condensate starts to fall under the influence of gravity and for some time interacts with the different components of the cavity field according to the Hamiltonian (4). While the zero photon component leads to a free time evolution of the condensate wavefunction, the single photon component imposes a periodic potential and thus scatters the atoms. Formally, equation (4) is equivalent to the interaction of atoms with a *classical* light field and we will therefore recover all the scattering phenomena well-known in that situation. In particular, for an appropriately designed interaction time and potential depth U_0 Bragg scattering will occur and the condensate will be totally reflected.

Note that this reflection of the condensate is performed by a single photon in the cavity and occurs on the same time scale as the scattering of a single atom. This can be viewed in two ways. First, in a semiclassical picture the photon simply creates an optical potential which all atoms feel simultaneously. Alternatively, one might think of the scattering of the condensate as a sequence of backscattering events of the photon by single atoms. Each of these processes imparts a recoil of $2\hbar k$ to one atom. However, because of our earlier assumption of adiabatically eliminated excited states of the atoms, the time during which the photon is absorbed by an atom is negligible. The photon is thus immediately re-emitted into the cavity mode, reflected by the cavity mirrors and can thus interact with all atoms virtually simultaneously. The total momentum of $2N\hbar k$ transferred to the atoms is thereby taken over by the cavity mirrors.

The total state of the system after the interaction of the atoms with the cavity (neglecting a global phase) is then given by the entangled state

$$\psi(t) = \frac{1}{\sqrt{2}}\left(|\hbar k\rangle \otimes |0\rangle + \mathrm{e}^{\mathrm{i}\alpha}|-\hbar k\rangle \otimes |1\rangle\right) \tag{7}$$

where the relative phase α of the two components arises from the different internal energies of the system during the interaction time. In the first component of the state ψ *all* of the atoms are in the same momentum state $|\hbar k\rangle$, whereas in the second component all atoms are in the state $|-\hbar k\rangle$. However, there is still information about the condensate state stored in the cavity, that is, a measurement of the cavity photon number will project the condensate wavefunction onto one of the momentum states $|\pm\hbar k\rangle$.

In order to destroy this "which path" information we may think of a measurement of the cavity state which projects onto one of the two orthogonal states $\psi_\pm = 1/\sqrt{2}(|0\rangle \pm |1\rangle)$. If the outcome of the measurement is $\psi_\pm$, the final state of the compound system will be

$$\psi_{\mathrm{f}} = \frac{1}{\sqrt{2}}\left(|\hbar k\rangle \pm \mathrm{e}^{\mathrm{i}\alpha}|-\hbar k\rangle\right) \otimes \psi_\pm \tag{8}$$

which is a product state of atomic and cavity degrees of freedom in both cases. The atomic part of this final state is a superposition state of all atoms going to one direction or to the other, that is, a Schrödinger cat state. Hence a measurement of the atomic position distribution sufficiently below the cavity will yield *all* atoms either at the left or at the right. However, recombining the beams later will nevertheless give rise to interference.

3 Rigorous treatment

The model which we used in the previous section to describe the basic principles and arising phenomena of the interaction of a condensate with a single cavity photon is of course highly idealised. We will thus review all the steps of this scheme in more detail in this section and discuss the required parameter regimes and limits. In particular, we will discuss under which conditions the atoms of an initial zero temperature condensate remain confined to a single quantum state.

3.1 Stochastic Schrödinger equation

In contrast to the previous section, we will now use a full many-particle treatment of the system and include cavity losses and a coherent pump for the cavity field. Since we are more interested in the outcome of a single run of the experiment rather than in ensemble averages, the appropriate theoretical framework is that of the stochastic Schrödinger equation (SSE) [22,23]. This provides us with the time evolution of the many particle and cavity wavefunction conditioned on the measurement results. In our case the SSE for the unnormalised wavefunction reads

$$\mathrm{d}|\Psi(t)\rangle = \left\{-\mathrm{i}\mathcal{H}_{\mathrm{eff}}\mathrm{d}t + \left(\frac{a}{\sqrt{\langle a^\dagger a\rangle}} - 1\right)\mathrm{d}N\right\}|\Psi(t)\rangle. \tag{9}$$

Here $\mathrm{d}N$ is a stochastic variable equal to one if a photon is detected in the time interval $[t, t+\mathrm{d}t]$ and zero otherwise. The effective many-particle and cavity Hamiltonian $\mathcal{H}_{\mathrm{eff}}$ is given by

$$\mathcal{H}_{\mathrm{eff}} = \int \mathrm{d}x\, \psi^\dagger(x) H_1(x,t)\psi(x) + \mathrm{i}\eta(a^\dagger - a) - \mathrm{i}\kappa a^\dagger a \tag{10}$$

where $\psi^\dagger(x)$ and $\psi(x)$ are the atomic field operators which obey bosonic commutation relations, η is the pump strength of the cavity, κ is the cavity decay rate, and H_1 is the single-particle Hamiltonian

$$H_1(x,t) = -\frac{\Delta_x}{2M} + U(x,t)a^\dagger a. \tag{11}$$

$$\begin{aligned} \mathrm{d}|\Psi(t)\rangle = \sum_m \frac{1}{\sqrt{N!m!}} \Big\{ -c_m(t)\kappa m \psi^\dagger[\phi_m(t)]^N + N c_m(t) \psi^\dagger[-\mathrm{i}H_1\phi_m(t)]\psi^\dagger[\phi_m(t)]^{N-1} \\ + c_{m-1}(t)\eta\sqrt{m}\psi^\dagger[\phi_{m-1}(t)]^N - c_{m+1}(t)\eta\sqrt{m+1}\psi^\dagger[\phi_{m+1}(t)]^N \Big\} (a^\dagger)^m|0,0\rangle \mathrm{d}t \\ + \sum_m \frac{1}{\sqrt{N!m!}} \Big\{ c_{m+1}(t) \frac{\sqrt{m+1}}{\langle a^\dagger a\rangle} \psi^\dagger[\phi_{m+1}(t)]^N - c_m(t)\psi^\dagger[\phi_m(t)]^N \Big\} (a^\dagger)^m|0,0\rangle \mathrm{d}N. \end{aligned} \quad (17)$$

All of the discussions in this work are independent of the detuning between the cavity and the pump field, since this only leads to additional relative phase shifts between the different photon number subspaces. For simplicity we have thus assumed in equation (10) that the pump laser is resonant with the optical cavity.

3.2 Preparation of the initial state

As before we assume that before the experiment starts N atoms form a zero temperature Bose-Einstein condensate confined in a trap which is placed slightly above the optical cavity. The cavity is assumed to be in its vacuum state. Given the condensate wavefunction ϕ_T in the trap, the full initial state reads

$$|\Psi\rangle = \frac{1}{\sqrt{N!}}\psi^\dagger[\phi_\mathrm{T}]^N|0,0\rangle \quad (12)$$

where $|0,0\rangle$ is the vacuum state of no atoms and no photons, and we have defined the atom creation operator for any normalised single-particle wavefunction $f(x)$ as

$$\psi^\dagger[f] = \int \mathrm{d}x\, \psi^\dagger(x) f(x). \quad (13)$$

Before releasing the atoms from the trap the cavity has to be prepared in the required initial state, ideally a superposition state of the form (5). This could be achieved in principle by the same technique which was successfully used in the microwave regime [24]. This exploits a two-level atom which is prepared in a superposition of ground state and excited state by a $\pi/2$ laser pulse. The atom then traverses the cavity thereby transferring the internal atomic state onto the cavity. In the optical regime, however, this method is hard to implement according to the short lifetime of the atomic excited state. A simple (though imperfect) alternative would be to prepare the cavity in a coherent state by use of the pump laser. For a pump strength η equal to the cavity decay rate κ, the steady state of the cavity (without interacting with atoms) is a coherent state of mean photon number one. The total state of the system at time $t = 0$ immediately after opening the atom trap then reads

$$|\Psi(t=0)\rangle = \frac{\mathrm{e}^{-1/2}}{\sqrt{N!}}\psi^\dagger[\phi_\mathrm{T}]^N \mathrm{e}^{a^\dagger}|0,0\rangle. \quad (14)$$

This state contains both the zero and one photon state with an equal amplitude of $1/\sqrt{\mathrm{e}} \approx 0.61$.

3.3 Cavity and atom interaction

After the time $t = 0$ the condensate is accelerated by gravity and for a certain time interacts with the cavity field according to the time dependent optical potential $U(x,t)$. The interaction time T is approximately given by $T = w/v_z$ with w being the cavity waist and v_z the transverse condensate velocity during the interaction. The assumption of negligible spontaneous atomic decay then reads $T < 1/(N\gamma)$, where $\gamma = \Gamma g^2/\Delta_a^2$ is the optical pumping rate (Γ is the atomic linewidth).

As we have already seen in Section 2, the time evolution of the atomic wavefunction depends on the cavity photon number and therefore entangles the cavity state and the atomic state. Assuming that within any subspace of fixed photon number m all atoms remain in a single wavefunction $\phi_m(t)$ for all times $t > 0$ leads to the ansatz

$$|\Psi(t)\rangle = \sum_m c_m(t)\frac{1}{\sqrt{N!m!}}\psi^\dagger[\phi_m(t)]^N(a^\dagger)^m|0,0\rangle \quad (15)$$

with the initial condition $\phi_m(t=0) = \phi_\mathrm{T}$ for all values of m. The coefficients $c_m(t)$ represent the time dependent amplitude of the m photon Fock state. Hence the time evolution of $|\Psi(t)\rangle$ is

$$\begin{aligned} \mathrm{d}|\Psi(t)\rangle = \sum_m \frac{1}{\sqrt{N!m!}} \big\{ \dot{c}_m(t)\psi^\dagger[\phi_m(t)]^N \\ + N c_m(t)\psi^\dagger[\dot{\phi}_m(t)]\psi^\dagger[\phi_m(t)]^{N-1} \Big\} (a^\dagger)^m|0,0\rangle \mathrm{d}t. \end{aligned} \quad (16)$$

On the other hand, applying the SSE (9) to the state (15) yields

see equation (17) above.

Comparing equations (16, 17) we see that in general the state of the system will *not* remain in the form of (15) during the time evolution. Hence the interaction of the condensate with a quantised cavity mode leads to heating of the atoms. There are two limiting cases, however, where this heating effect can be neglected. First, if the cavity state is a coherent state of high intensity (standard deviation of the photon number much smaller than the mean photon number), the atomic dynamics becomes effectively that of an interaction with a well defined field intensity. In this case there exists a unique atomic wavefunction for all photon number subspaces. Thus the total state of the system at any time is a product state of the atomic state, where all atoms occupy this unique wavefunction, and a coherent cavity state. This semiclassical limit has been investigated in detail in reference [19].

The second case which is consistent with equation (15) is that of a vanishing pump strength $\eta = 0$, meaning that the driving laser has to be switched off before the condensate enters the cavity mode. Note that, in principle, a finite cavity decay rate κ does not destroy the form (15) of the wavefunction. However, the non-Hermitian time evolution in this case changes the relative amplitude of the various photon number states even if no actual quantum jump occurs. Hence the cavity will no longer act as a 50–50 beam-splitter even for the perfect initial cavity state (5). On the other hand, if a photon is detected at time t, the corresponding quantum jump will change the relative amplitudes of the photon number states *and* will change the partial wavefunctions by $\phi_m(t+\mathrm{d}t) = \phi_{m+1}(t)$. According to the random time at which such a jump occurs, it is no longer possible in this case to obtain a well-defined final state after the interaction of the condensate with the cavity by a proper adjustment of the system parameters, such as interaction time and optical potential depth. Finally, for the initial state (5) a quantum jump is tantamount to a complete measurement of the cavity state and thus leaves the system in a well defined state without cat-like coherence.

Therefore, although quantum jumps do not in principle destroy the general structure (15) of the atomic wavefunction, cavity decay should be suppressed for the sake of controlability of the final state. Moreover, in order to obtain full knowledge of the state of the system, photons leaving the cavity need to be detected with unit efficiency. This implies that the interaction time T must be short compared to $1/\kappa$.

Now let us compare this with the interaction time which is necessary to achieve complete Bragg reflection. In the Bragg regime an atom will be scattered from an initial state with momentum $\hbar k$ into a final state $-\hbar k$ without significant population of other momentum states. Therefore the required energy uncertainty is of the order of the recoil energy $\hbar\omega_{\mathrm{R}}$, $\omega_{\mathrm{R}} = \hbar k^2/(2M)$, and *via* the Heisenberg uncertainty relation we obtain the Bragg condition $T > 1/\omega_{\mathrm{R}}$. Observation of Bragg reflection in our system thus implies a cavity decay rate of the order of the recoil frequency, $\kappa \approx \omega_{\mathrm{R}}$. Considering the case of lithium (the lightest alkali atom for which condensation has been achieved experimentally) and the parameters of the high-finesse cavities used in the single-atom experiments [25,26], the cavity lifetime has to be increased by about a factor of 20 which can be obtained by increasing the cavity length by this factor. Accordingly the mode volume increases by a factor of $20^{3/2}$ and hence the cavity-atom coupling strength is reduced by one order of magnitude. Using the conditions for small atomic decay and small cavity decay $\kappa, \gamma \approx \omega_{\mathrm{R}}$ (for a single atom) finally gives an optical potential depth $U_0 \approx 2.4\omega_{\mathrm{R}}$. Numerical integration of the SSE (without pump) shows that this value is approximately a factor of 5 to 10 too small to achieve Bragg reflection within the interaction time T. Hence, although the ideal situation of complete Bragg reflection with a single photon is not experimentally feasible at the moment, it might become possible within the next few years.

However, for the preparation of a cat-like state, Bragg reflection is not necessary. In the absence of the driving laser the dynamics guarantees that within any m photon number subspace all atoms populate the same wavefunction $\phi_m(t)$ at all times. Without fulfilling the Bragg condition, the final states $\phi_m(t)$ after the atom-cavity interaction will in general not be orthogonal. In other words, detecting a single atom in the initial momentum state $\hbar k$ after the interaction will not lead to a collapse of all atoms into this state. On the other hand, if the cavity was initially prepared in a superposition of the zero and one photon state and a single atom is detected in a momentum state different from the initial one, all other atoms will collapse into the wavefunction $\phi_1(t)$ and a cat-like behaviour is recovered.

3.4 Measurement of the cavity state

After the interaction of the condensate with the cavity field, the system is thus in an entangled state of form (15). Therefore a measurement of the cavity photon number will give precise information on the state of the condensate. For example, cavity decay will project the atomic superposition state to a single component. It is thus necessary to erase the information stored in the cavity. We will discuss several possibilities how to perform this in the following.

As we have shown in Section 2, the ideal measurement of the cavity state for this purpose would be one which projects onto one of the orthogonal states $\psi_\pm = 1/\sqrt{2}(|0\rangle \pm |1\rangle)$. In this case the state of the system after the measurement is a product state (8) of atomic state and cavity state and thus the subsequent atomic dynamics is decoupled from the cavity dynamics. However, a measurement of this kind is difficult to realise experimentally.

Another possibility is to detect the cavity state by a homodyne measurement, that is, mixing the light transmitted through the cavity with a strong laser beam. However, in general after the interaction with the condensate the cavity is not in a coherent state even if it was initially. Homodyning will thus add a lot of noise to the system. Our numerical simulations show that starting from the ideal state, equation (7), after homodyne detection the final state has a probability of being in the Bragg scattered beam with a nearly uniform distribution in the interval $[0, 1]$. Therefore, this procedure in principle creates an atomic cat state but with a random amplitude of the two components.

A third way to erase the cavity state is to switch on the driving laser again after the condensate has left the cavity mode. This mixes the cavity state with the coherent state of the laser and thereby information is destroyed. Together with spontaneous cavity decays this gives rise to a final product state of atomic and cavity degrees of freedom. Although this method is experimentally easy, it suffers from the same problem as the homodyne detection scheme, that is, according to the random times of the cavity decays the final atomic state has an unknown relative amplitude of the two momentum components.

Instead we propose another method which consists of a short pulse of the driving laser and postselection of the final state on the zero photon cavity state. This leads to a well-defined final state with a sufficiently high efficiency as we will show in the following.

Suppose that after the condensate-cavity interaction the system is in a state of form (15). Then the driving laser is switched on for a time τ which is short compared to the cavity decay time $1/\kappa$. In general this implies that τ also is short compared to the time scale of the dynamics of the atomic degrees of freedom, which is of the order of $1/\omega_{\mathrm{R}}$. The Hamiltonian (10) thus reduces to

$$\mathcal{H}_{\mathrm{eff}} = \mathrm{i}\eta(a^\dagger - a) \tag{18}$$

and the system state after the laser pulse is given by

$$\begin{aligned}|\Psi(t+\tau)\rangle &= \mathrm{e}^{-\mathrm{i}\mathcal{H}_{\mathrm{eff}}\tau}|\Psi(t)\rangle \\ &= \mathrm{e}^{-(\eta\tau)^2/2}\sum_m c_m(t)\frac{1}{\sqrt{N!m!}}\psi^\dagger[\phi_m(t)]^N \\ &\quad \times \mathrm{e}^{\eta\tau a^\dagger}(a^\dagger - \eta\tau)^m|0,0\rangle. \end{aligned} \tag{19}$$

Projecting onto the zero photon subspace, which is tantamount to a postselection on the case where no cavity decay photons are detected, yields the final state

$$|\Psi_{\mathrm{f}}\rangle = \mathrm{e}^{-(\eta\tau)^2/2}\sum_m c_m(t)\frac{1}{\sqrt{N!m!}}\psi^\dagger[\phi_m(t)]^N(-\eta\tau)^m|0,0\rangle \tag{20}$$

the squared norm of which gives the probability of finding the desired zero photon state. As an example let us assume that the system is in state (7) before the final laser pulse and $\tau = 1/\eta$. Then the final state reads

$$|\Psi_{\mathrm{f}}\rangle = \frac{\mathrm{e}^{-1/2}}{\sqrt{2N!}}\left\{\psi^\dagger[\phi_0(t)]^N - \mathrm{e}^{\mathrm{i}\phi}\psi^\dagger[\phi_1(t)]^N\right\}|0,0\rangle. \tag{21}$$

This is a perfect Schrödinger cat state with a squared norm of 1/e. The probability for this method of being successful is thus 37%. Note, however, that this scheme demands the detection of spontaneously emitted photons with unit efficiency. For imperfect detection the final state of the system will be an incoherent mixture of a cat state with other superposition states of different wave functions and different relative amplitudes. In this case the proposed system will fail with a certain probability to create the desired cat state. A similar effect occurs, if there is a finite probability for photon absorption in the cavity mirrors.

4 Numerical examples

In this section we study a specific numerical example of the system proposed in the previous section. The parameters used here are those for $N = 10$ rubidium atoms and the optical cavity used by Rempe and coworkers [25], with the only idealisation that we increase the optical potential U_0 by a factor of ten.

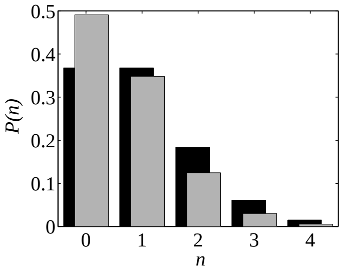

Fig. 1. Cavity photon number distribution $P(n)$ after the preparation laser pulse, step (ii), (black bars), and after the condensate-cavity interaction time, step (iii), (grey bars). See text for details and choice of parameters.

Therefore we use a cavity decay rate of $\kappa = 400\omega_{\mathrm{R}}$. Allowing for a certain probability of cavity decays during the atom-cavity interaction, we set $2\kappa T = 0.2$ and hence $T = 2.5\times10^{-4}\omega_{\mathrm{R}}^{-1}$. The probability of spontaneous atomic decay is set to the same order of magnitude by the condition $N\gamma = 2\kappa$. This allows us to calculate the appropriate value for the atom-light detuning and then the optical potential depth $U_0 = 13\,280\omega_{\mathrm{R}}$. The experiment which we simulate proceeds in the following steps.

(i) The condensate is prepared initially in a trap which is situated slightly above the cavity. The spatial size of the condensate wavefunction is much larger than the cavity wavelength, such that the momentum width is small compared to the photon momentum $\hbar k$. The trap is switched of and the condensate falls due to gravity. The cavity is initially in its vacuum state.

(ii) When the condensate enters the cavity, more precisely at position $x = -w$, a laser pulse is used to prepare the cavity field in a coherent state. The length τ of this pulse is assumed to be much shorter than the cavity decay time and the laser intensity is chosen such that $\eta\tau = 1$. Hence the mean cavity photon number after the preparation pulse is one. The photon number distribution at this time is shown by the black bars in Figure 1.

(iii) Then the condensate interacts with the cavity for the time T. During this time, cavity decays and atomic spontaneous emissions may occur according to the decay rates κ and $N\gamma$. If such an incoherent event happens, the result of this experimental run is disregarded and the experiment is repeated. For our parameters, the simulations show that a quantum jump occurs with a probability of approximately 0.25.

The transverse velocity of the condensate is chosen such that after the interaction time T the condensate position is $x = w$. Due to the different decay probabilities for different photon numbers, the cavity photon number distribution changes during the time T. The final distribution is shown by the grey bars in Figure 1. Hence, although the cavity was prepared with equal amplitudes for the zero and one photon states, the final state comprises a higher amplitude for the zero photon state. According to the different potential depths, the atomic wavefunctions ϕ_n conditioned on the cavity photon number n are scattered differently within the time T. In Figure 2 we

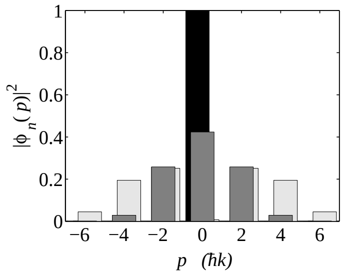

Fig. 2. Momentum distribution of the partial wavefunctions ϕ_n conditioned on the cavity photon numbers $n = 0$ (black bars), $n = 1$ (dark grey), and $n = 2$ (light grey) after the interaction of the condensate with the cavity. See the description of step (iii) in the text for details.

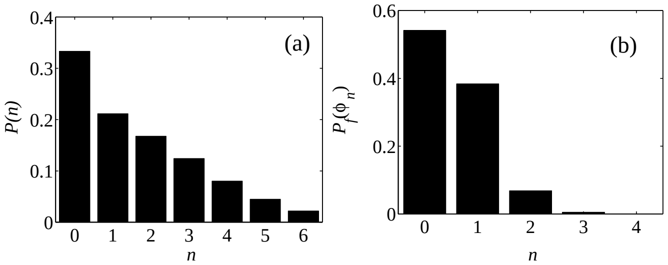

Fig. 3. (a) photon number distribution in the cavity after the second laser pulse, step (iv) in the text. (b) Probabilities of the wavefunctions ϕ_n in the final state $|\Psi_\mathrm{f}\rangle$. See text for details.

show the final momentum distributions for the three lowest wavefunctions, $m = 0...2$. For the chosen parameters, the scattered (single particle) wavefunction ϕ_1 has a significant overlap with ϕ_0, whereas ϕ_2 and ϕ_0 are essentially orthogonal. Note, however, that the overlap of the N particle wavefunctions scales as

$$\left|\frac{1}{N!}\langle 0,0|\psi[\phi_1]^N\psi^\dagger[\phi_0]^N|0,0\rangle\right|^2 = |(\phi_1,\phi_0)^N|^2, \qquad (22)$$

where $(.,.)$ denotes the single particle scalar product. The overlap of the N particle wavefunctions is thus of the order of 0.001 for our parameters. By increasing the optical potential U_0 by approximately another factor of two it can be achieved that the wavefunction ϕ_1 has no zero momentum component instead of ϕ_2. In this case, the final Schrödinger cat state consists essentially of two *orthogonal* single-particle wavefunctions. However, in the following we will stick to the more general case of a finite overlap of the wavefunctions.

(iv) Another laser pulse is applied to the cavity with the same properties as the pulse in step (ii). The photon number distribution after this step is shown in Figure 3a.

(v) Finally, the system state is projected onto the zero photon subspace. Experimentally this is performed by simply waiting for several cavity decay times. If a cavity photon is detected during this time, the experimental run is again disregarded, otherwise the required atomic state is prepared. The probability of finding the zero photon state is approximately 1/3 according to Figure 3a.

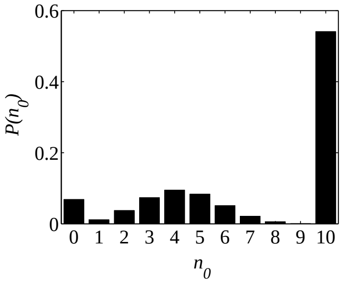

Fig. 4. Probability distribution $P(n_0)$ of detecting n_0 atoms in the state ϕ_0. The two peaks at $n_0 = 0$ and $n_0 = 10$ occur due to the ϕ_2 respectively ϕ_0 component of the Schrödinger cat state, the binomial distribution around $n_0 = 4$ due to the state ϕ_1 which has a nonzero overlap with ϕ_0.

Together with the 75% chance that no photon was detected during the interaction time T, see step (iii) above, we obtain an overall efficiency of 0.25. Thus, in one out of four experimental runs we will get a cat-like state of the condensate. Figure 3b shows the probabilities

$$P_\mathrm{f}(\phi_n) = \left|\langle\Psi_\mathrm{f}|\frac{1}{\sqrt{N!}}\psi^\dagger[\phi_n]^N|0,0\rangle\right|^2 \qquad (23)$$

of finding the states ϕ_n in this final state. According to the initially prepared coherent state instead of state (5) and the different relative decay amplitudes during the interaction time, the final state is not a 50–50 superposition state of ϕ_0 and ϕ_1. However, we obtain numerically a propability of 54% that all atoms are in state ϕ_0 and of 38% that all atoms are in state ϕ_1.

It should be emphasised once more that in general the wavefunctions ϕ_n are not mutually orthogonal. For instance, in the numerical example considered here the wavefunction ϕ_1 has a relatively large component of the initial zero momentum eigenstate, see Figure 2, and therefore a significant overlap with ϕ_0. Hence, if the final state is measured by counting the number of atoms n_0 in state ϕ_0, the resulting atom number distribution has three distinct components, *cf.* Figure 4: first, a peak at $n_0 = 10$ corresponding to the case where all atoms are in state ϕ_0. Second, a binomial distribution with a maximum at $n_0 = 4$ coming from the component where all atoms are in state ϕ_1. Third, if all atoms are in the state ϕ_2 which has no zero momentum component (see Fig. 2), no atoms are detected in ϕ_0 which gives rise to the peak at $n_0 = 0$. This multiply peaked atom number distribution is a good indication for the cat-like nature of our final condensate state since such a distribution cannot be achieved, for instance, by scattering the condensate by a standing wave laser beam, where one just gets a binomial distribution.

In a second numerical example, Figure 5, we chose parameters such that the Bragg condition is fulfilled and a condensate of $N = 10$ lithium atoms is nearly completely reflected by a single cavity photon. This requires an atom-cavity interaction time $T = 1/\omega_\mathrm{R}$ and thus a cavity decay rate $\kappa = \omega_\mathrm{R}/10$. Compared to our previous example one thus has to increase the length of the cavity by a factor

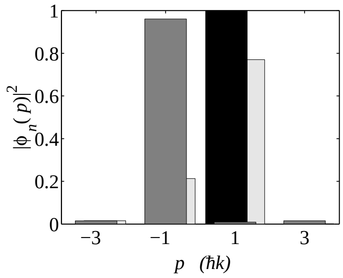

Fig. 5. Bragg reflection of Li atoms: momentum distribution of the partial wavefunctions ϕ_n conditioned on the cavity photon numbers $n = 0$ (black bars), $n = 1$ (dark grey), and $n = 2$ (light grey). Parameters are $N = 10$, $T = 1/\omega_R$, $\kappa = \omega_R/10$, $U_0 = 8.5\omega_R$.

of about 240 (which gives a length of 2.8 cm [25]). The calculated optical potential then is two orders of magnitude too small. Instead we use a value of $U_0 = 8.5\omega_R$ here. Additionally, we now assume that the atoms are initially in a momentum eigenstate of one photon momentum $\hbar k$ (black bar in Fig. 5). The parameters are chosen in such a way that all effects related to the cavity pump field and to spontaneous atomic or cavity decay are very close to those presented in Figures 1 and 3. We thus concentrate on the momentum distributions of the conditional wavefunctions after the interaction of the condensate with the cavity, Figure 5. We see that the wavefunction conditioned on one cavity photon ϕ_1 has a negligible component of the $+\hbar k$ momentum state, but a probability of 0.96 for the $-\hbar k$ state. The final state $|\Psi_f\rangle$, equation (20), is then to a good approximation a state where all atoms are either in the $+\hbar k$ or in the $-\hbar k$ state with a small admixture of the state ϕ_2 which contains both components. An atom number distribution as in Figure 4 will thus show two sharp peaks at $n_0 = 0$ and $n_0 = 10$ indicating a highly non-classical state of the atoms.

5 Conclusions

We have investigated in detail the interaction of an ideal condensed Bose gas with a quantized mode of an optical cavity in the strong coupling regime. In the limit of a large detuning of the light frequency from the atomic resonance, scattering of the condensate works analogously to scattering of a single atom and, in particular, on the same time scale. Nevertheless there are some important limitations if the coherence of the condensate is to be preserved during the scattering process. We find that coherent pumping of the cavity field during the interaction with the condensate destroys the condensate coherence, while spontaneous cavity decay in principle does not lead to heating of the condensate but limits the experimental precision. Hence, for controlled manipulation of the condensate by one or a few photons it is desirable to avoid spontaneous cavity decay.

We have used these properties of the condensate and cavity interaction to propose a method of preparing a mesoscopic superposition, a so-called Schrödinger cat state, of the external degrees of freedom of the condensate. From our discussion of experimental parameters we conclude that an improvement of cavity parameters by only one order of magnitude as compared to published values will allow the transition from a gedanken experiment to a real one.

For simplicity our discussions here were based on an ideal Bose-Einstein condensate as the initial state of the atoms. However, weak atom-atom interactions would not qualitatively change the properties of the system. In this case the details of the scattering process such as required interaction times, amplitudes of scattered beams, and atomic wavefunctions would be altered, but the entanglement of the atomic state and the cavity photon number would persist. In principle even a thermal atomic beam could be used in such a setup. In this case one would have to ensure that a single photon scatters the individual atoms by more than the width of the initial momentum distribution. This is possible in principle either by a narrow velocity filter, as for example in the experiments by Rempe and coworkers [25], or otherwise by using short interaction times and deep optical potentials (the Raman-Nath regime of scattering theory). The resulting Schrödinger cat state would then consist of two beams which are defined by different ranges of momenta, rather than by definite wave functions as in the case of a condensate. However, experimentally it would then be hard to achieve high enough atomic densities and sufficiently long cavity life times which are required for the suppression of decoherence during the interaction time of the atoms and the cavity field.

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